The van Hiele Levels of Geometric Thought
(adapted from: *Teaching Elementary and Middle School Mathematics: Teaching Developmentally, 4th Edition*, by John A. van de Walle. Addison-Wesley Longman publishers)

Level 0: Visualization
*The **objects of thought** at level 0 are shapes and what they "look like."*
*The **products of thought** at level 0 are classes or groupings of shapes that seem to be alike.*

Level 1: Analysis
*The **objects of thought** at level I are classes of shapes rather than individual shapes.*
*The **products of thought** at level I are the properties of shapes*

Level 2: Informal Induction
*The **objects of thought** at level 2 are properties of shapes.*
*The **products of thought** at level 2 are relationships among properties of geometric objects.*

Level 3: Deduction [note: intended for high school]
*The **objects of thought** at level 3 are relationships among properties of geometric objects.*
*The **products of thought** at level 3 are deductive axiomatic systems for geometry.*

Level 4: Rigor
*The **objects of thought** at level 4 are deductive axiomatic systems for geometry.*
*The **products of thought** at level 4 are comparisons and contrasts among different axiomatic systems of geometry.*

Characteristics:
- Levels are sequential. Generally a level cannot be skipped. In this way this theory is similar to developmental stage theories, such as Piaget's.
- Levels are not age-dependent. (Here is where level theories differ from stage theories.) Increased experiences increase the likelihood that people develop beyond level 0 with adequate instruction.
- Advancement through the levels is by and large promoted through carefully chosen geometric experiences.
- Instruction must be at the level at which the student(s) currently operate(s).
- It is the type of thinking that children are required to do that makes the difference in learning, not the specific content. In other words, the same activity can stimulate thought at different levels (i.e. differentiating instruction by addressing levels of thinking within the same task).

For K-8 we are generally concerned with levels 0 through 2. What are appropriate features of activities at these three levels?

Level 0:

- Involve lots of sorting, identifying, and describing of various shapes
- Use lots of physical models that can be manipulated by the students
- Include many different and varied examples of shapes so that irrelevant features do not become important. (Many students believe that only equilateral triangles are really triangles or that squares turned 45 degrees are no longer squares.)
- Provide opportunities to build, make, draw, put together, and take apart shapes.

Level 1:

- Begin to focus more on properties of figures and objects than on simple identification (description). Define, measure, observe, and change properties with the use of models.
- Use problem-solving contexts in which properties of shapes are important components.
- Continue to use models, as with level 0, but include models that permit the exploration of various properties of figures.
- Classify figures based on properties of shapes as well as by names of shapes. For example, find different properties of triangles that make some alike and others different.

Level 2:

- Continue to use models with a focus on defining properties. Make property lists, and discuss which properties are necessary and which are sufficient conditions for a specific shape or concept.
- Include language of an informal deductive nature: *all*, *some, none, if-then, what if*, and the like.
- Investigate the converse of certain relationships for validity. For example, the converse of "If it's a square, it must have four right angles" is "If it has four right angles, it must be a square."
- Use models and drawings as tools to think with and begin to look for generalizations and counterexamples.
- Encourage the making and testing of hypotheses.

It is important to distinguish between objectives that address geometric content (concepts, terminology and notation) and those that address geometric growth (spatial sense and geometric reasoning) and to address both in curriculum and instruction. The "grand" objective in grades K-8 is to move children from being "level 0 thinkers" to being "level 2 thinkers."

What do we need to look for in the transition from level 0 to level 1 and in the transition from level I to level 2?

Level 0 --> Level 1:
At level 0, students are restricted in thought to the shapes they are currently working with, while at level 1, students attribute properties to classes of shapes (e.g., all rectangles).

- Child attends to a variety of characteristics of shapes in sorting and building activities.
- Child uses language that is descriptive of geometric shapes
- Child shows evidence of geometric reasoning in solving puzzles, exploring shapes, creating designs, and analyzing shapes.
- Child recognizes shapes in the environment.
- Child solves spatial problems.

Level 1 --> Level 2:
Most students will not have reached level 2 thinking by grade 6 or 7 according to Van de Walle. Level 1 thinkers are exclusively thinking inductively, while level 2 thinkers begin to think deductively. When we help children move toward level 2 they will increasingly demonstrate the following:

- Child shows improvement in spatial visualization skills.
- Child has an inclination to make and test conjectures in geometric situations.
- Child makes use of logical explanations in geometric problem solving.
- Child justifies conclusions in geometric contexts.
- Child assesses the validity of logical arguments in geometric situations.